Diffusion Induced Pattern Formation in a Minimal Sediment Model

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Motivation:

The degradation of chemical substances in the sediment is performed by a complex network of interacting populations of microorganisms. To study the emergence of spatial patterns in the sediment, we analyze a part of the relevant processes, in particular the interaction between degradation and transport processes in a reduced model (minimal sediment model or MS-model).

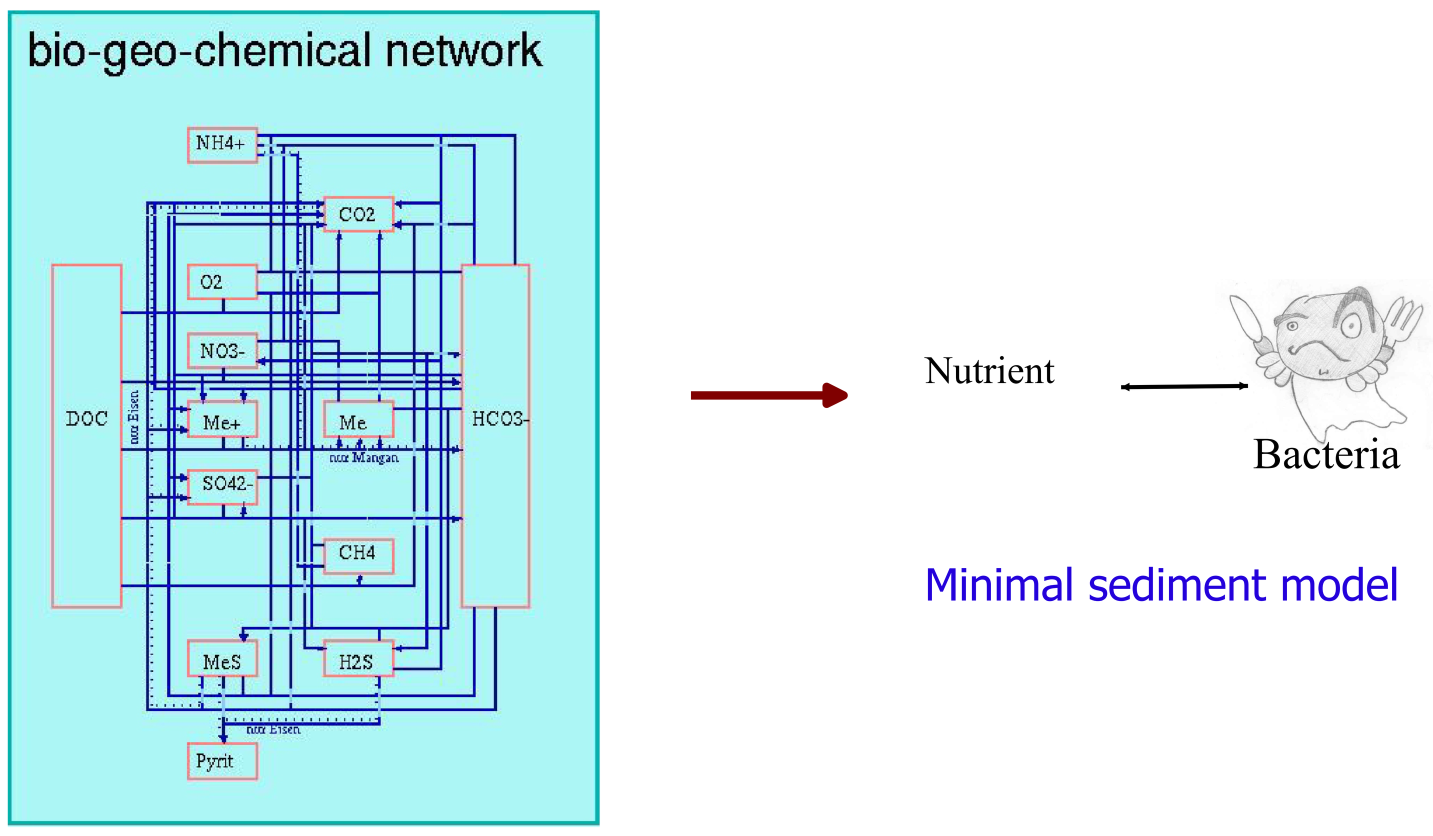

Figure 1: Instead of the complex network of bacteria-substrat intercations, we consider a minimal sediment model.

Setup of the Model:

We employ the MS-model on a one-dimensional domain, regarding the processes shown in Fig.2. Thus the model is governed by the Equations shown in Tab.1.

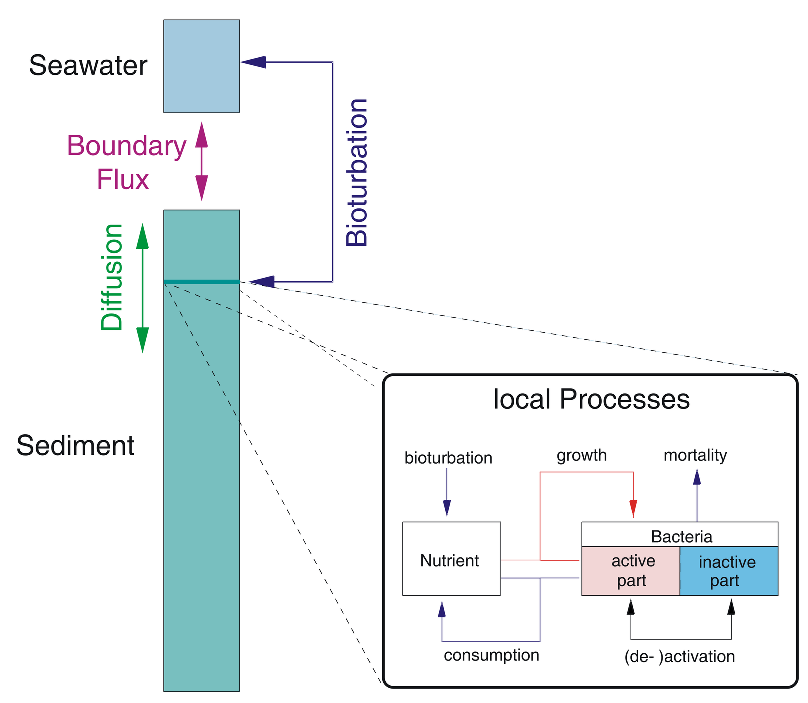

Figure 2: Processes considered in the MS-model

Additional to the equations boundary conditions have to be specified. We generally employed a no-flux condition at the boundaries, except at the upper one, where we allow a flux of nutrient flowing into the system.

$$\frac{\partial X}{\partial t} = \boxed{\frac{X}{X+K}XY} - \boxed{mX} + \boxed{D_X\frac{\partial^2 X}{\partial z^2}}$$

$$\frac{\partial Y}{\partial t} = \boxed{\Phi\exp(-az)\Big(Y_0 - Y\Big)} - \boxed{\frac{X}{X+K}XY} + \boxed{D_Y\frac{\partial^2 Y}{\partial z^2}}$$

Table 1: The balance equations for the variables X and Y consist of the contributions stemming from the different processes: Degradation (red), bioirrigation (blue), loss of bacteria (black) and diffusion (green).

Results (local model):

To study the dynamical properties of the MS-model, firstly we analyze the temporal behaviour of the local model. In a local formulation all spatial effects are neglected, so that the model is employed on an isolated point. We find that there always exists a trivial equilibrium state at (0,Y0) with extinct population of bacteria. It can be accompanied by a pair of non-trivial equilibria E1=(X1,Y1) and E2=(X2,Y2) with 0<X1<X2 and Y2<Y1<Y0 . E1 is always a saddle, whereas E2 is either attracting or repelling. In the latter case, the stability characteristics of E2 can be changed by parameter variation: A sufficiently high increase of Y0 , for instance, will have the result, that E2 will turn into an attractor. When changing its characteristics E2 traverses a subcritical Hopf bifurcation and an unstable periodic orbit emerges (see Fig.3).

Results (local model):

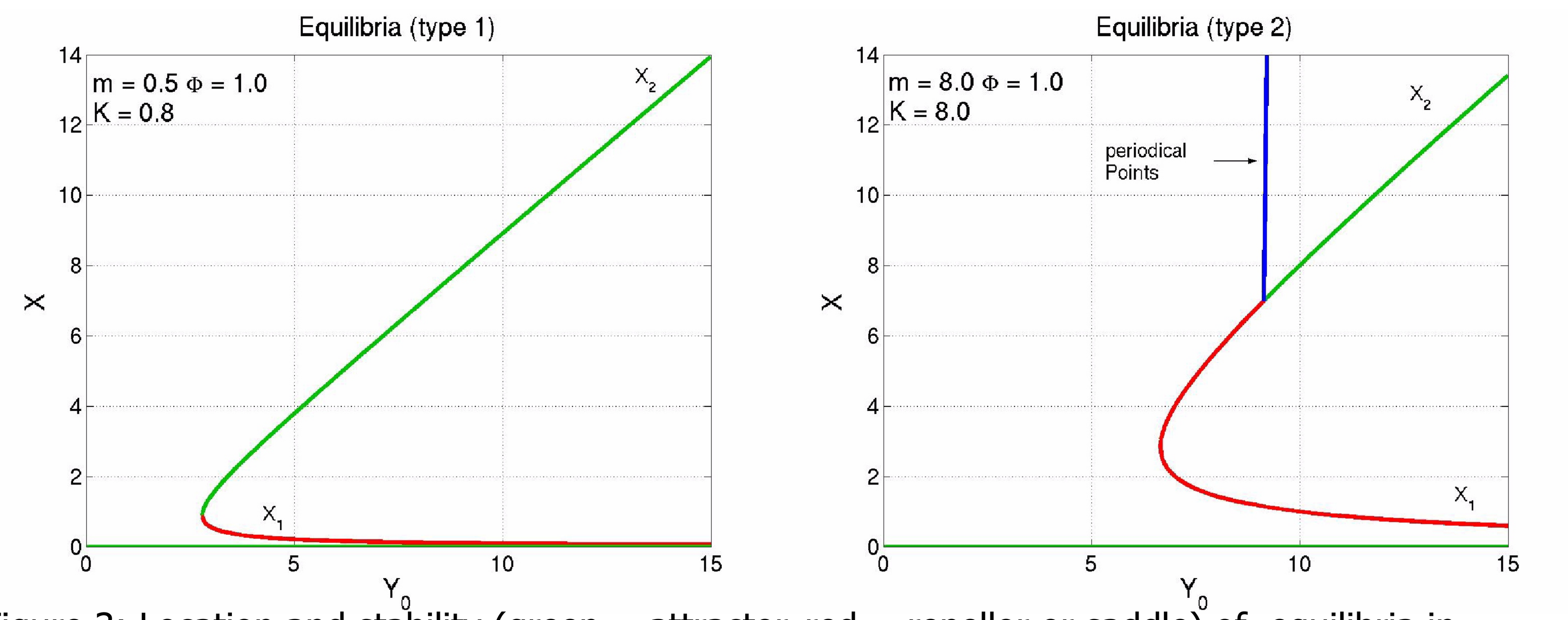

Figure 3: Location and stability (green = attractor, red = repellor or saddle) of equilibria in the local model.

Results (spatial model):

Numerical experiments show, that the long-term behaviour of the spatial MS-model is characterized by constant profiles (composed of X2-states), if diffusion is neglected (see Fig.4, timesteps 0-3).
Certain diffusion effects evoke the appearence of so-called Turing instabilities: The constant profiles are loosing their stability and spatial patterns appear instead (see Fig.4).

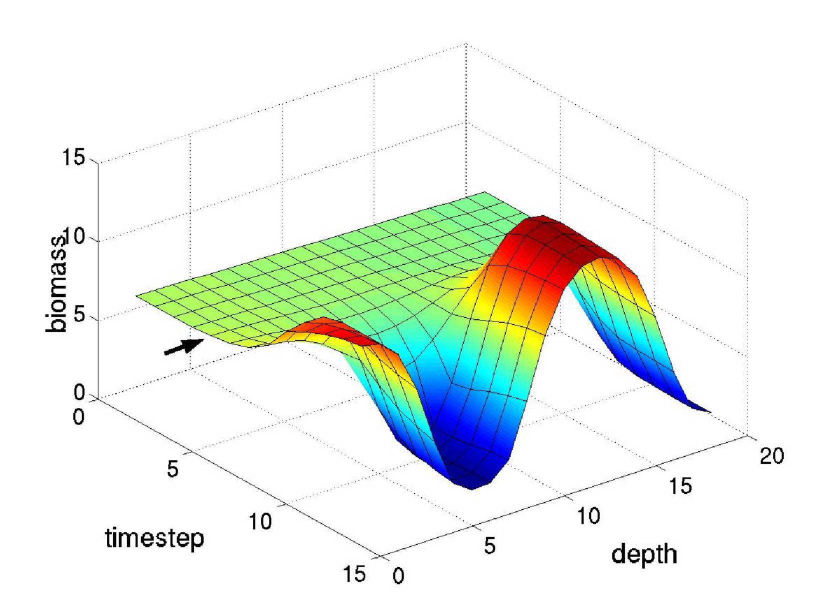

Figure 4: profiles of the MS-model in a timeserie. Starting with a constant profile, we add a small perturbation at timestep 4 (arrow). The system departs from the monotonous state and forms a oscillating structure instead.

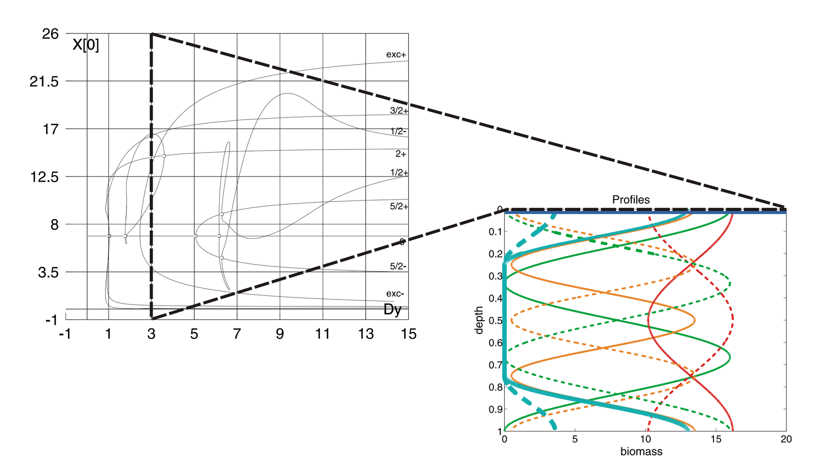

Figure 5: The concept of a bifurcation diagram: To illustrate, how profiles change with a parameter (here Dy), the dis-tributions are evaluated for some values of Dy. At a certain depth the values of the profiles are recorded and transferred into the bifurcation diagram.

The bifurcation diagrams in Fig. 6 show, that for small values of Dy the only equilibrium state is the non-diffusive one (0-profile). When Dy is increased, several profiles accompany the 0-profile. Further analysis shows, that the 0-profile looses stability and other distributions (possibly more than one at one time) become attracting.

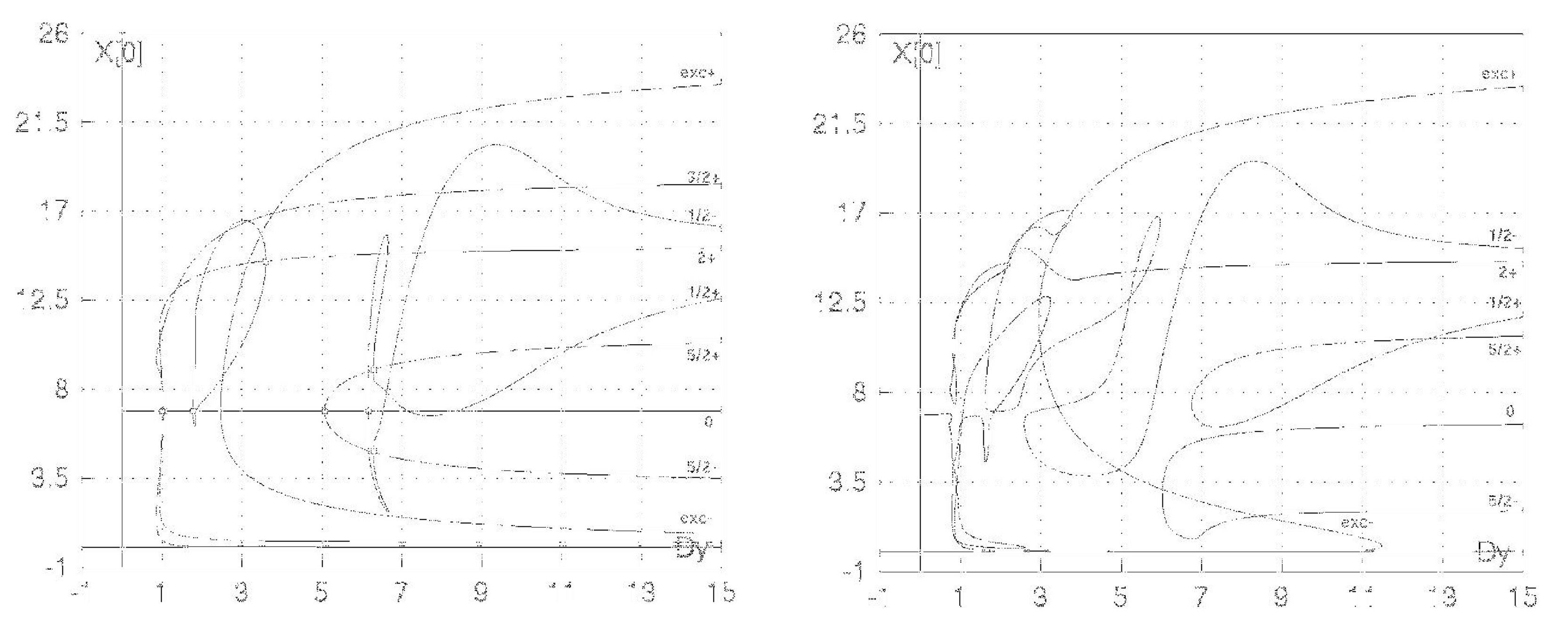

Figure 6: Bifurcation diagrams with constant bioirrigation (left) and decreasing bioirrigation (right). Squares symbolize bifurcations (not all plotted).

Conclusions:

The preconditions of pattern formation can be summarized as follows:

(1) The diffusion of nutirent is high in comparison to that of the bacteria. Considering the different sizes of bacteria and ntrients and taking into account that bacteria tend to stick on the sediment matrix, this condition is reasonable.
(2) The growth of bacteria is autocatalytic. This special feature follows from the activation mechanism of the MS-model.
(3) The input of nutrient into the system is high in comparison to the loss of bacteria.

Under such conditions the emergence of inhomogeneous spatial patterns takes place without any external forcing.