On the Impact of Interaction Functions on Foodchain Stability

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Motivation

- The dynamics of models should not depend strongly on the exact mathematical form of the model equations. The "real" functions may only be determined with a certain accuracy and may change in time because of processes like adaptation.
- We discuss the effect of variation of the functional form of species interaction in general foodchain models.
- The methods of bifurcation analysis (Guckenheimer et al. 97, Gross and Feudel 03) are used. Our results do not depend on a certain model but are valid for a large class of models.

Conclusions

- Stability properties of the model depend on the exact functional form of the interaction functions.
- The stability of mathematically simple interaction functions (e.g. Holling functions) decreases monotonously with prey abundance. Models, in which these functions are used, are destabilized by enrichment.
- Interaction functions exist which are similar in shape but have dramatically different stability properties, including large intervalls in which enrichment is stabilizing.

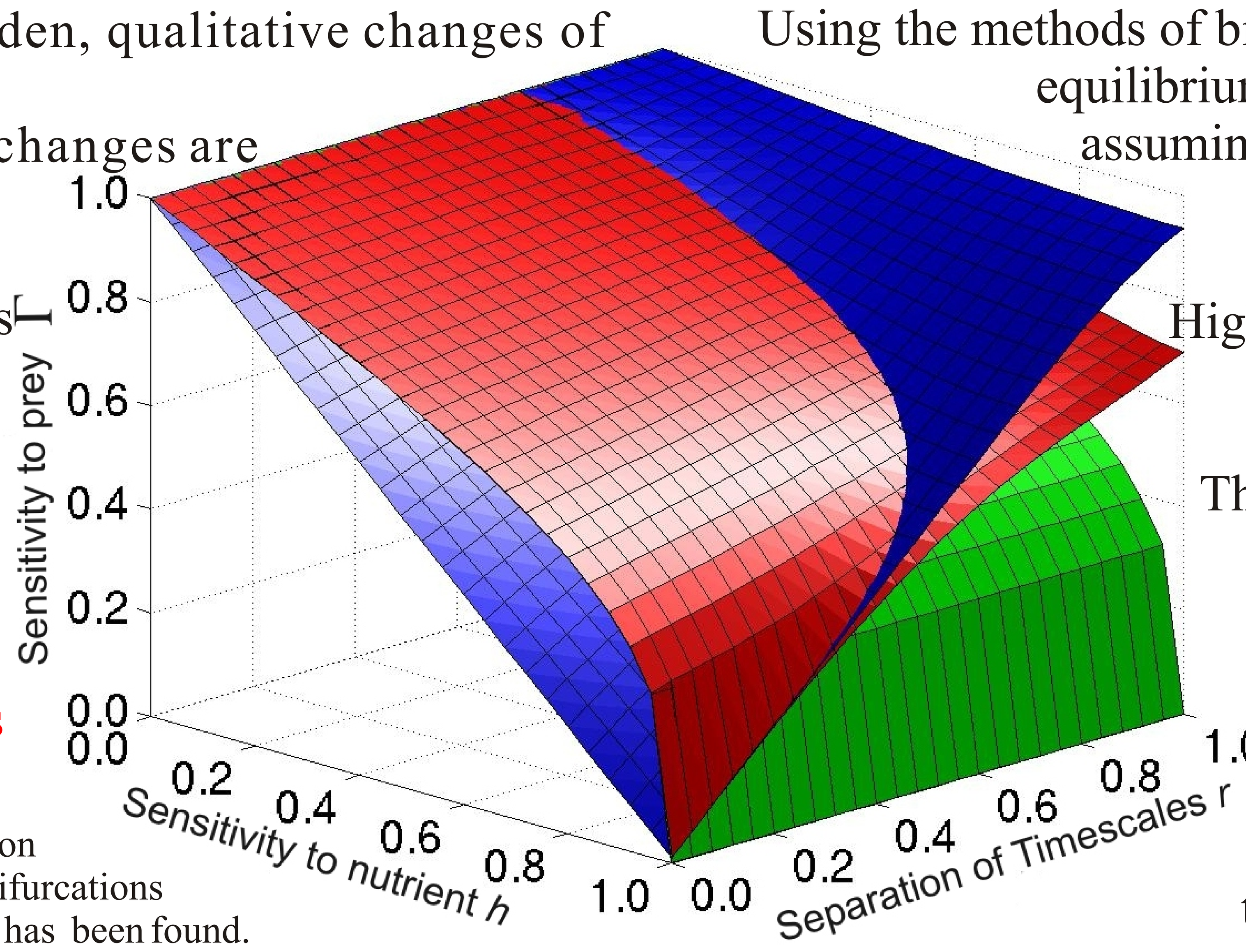
Bifurcations in General Foodchains

If model parameters are changed sudden, qualitative changes of the long-term dynamics may occur. Such changes are called bifurcations.

A stable equilibrium which undergoes a bifurcation becomes unstable.

In our foodchain model the primary loss of stability occurs because of Hopf bifurcations.

Hopf bifurcations indicate transitions from stationary to periodic dynamics.



Center of page: Bifurcation diagram of a five-trophic foodchain. The equilibrium is stable above the bifurcation surfaces. Stability is lost by crossing one of the Hopf bifurcations (red, blue). In addition a transcritical bifurcation (green) has been found.

Below: Some commonly used interaction functions G(X) and the corresponding stability functions. χ =X K

Function	$G(X)$	$\Gamma(\chi)$
Lotka-Volterra	AX	1
General Holling	$\frac{AX^\alpha}{K^\alpha+X^\alpha}$	$\frac{\alpha}{1+\chi^\alpha}$
Multiple Saturation	$AX^M\prod_{m=1}^M(K_m+X)^{-1}$	$\sum_{m=1}^M(\chi_m+1)^{-1}$
Gamov	$A\exp(-K/X)$	$1/\chi$
Ivlev	$A(1-\exp(X/K))$	$1+\frac{(\chi+1)\exp(-\chi)}{1-\exp(-\chi)}$
Power law	AX^α	$\frac{1}{\alpha}$

The Impact of Interaction Functions

Using the methods of bifurcation analysis, the stability of equilibrium states can be evaluated without assuming any specific functional form for predator-prey interactions.

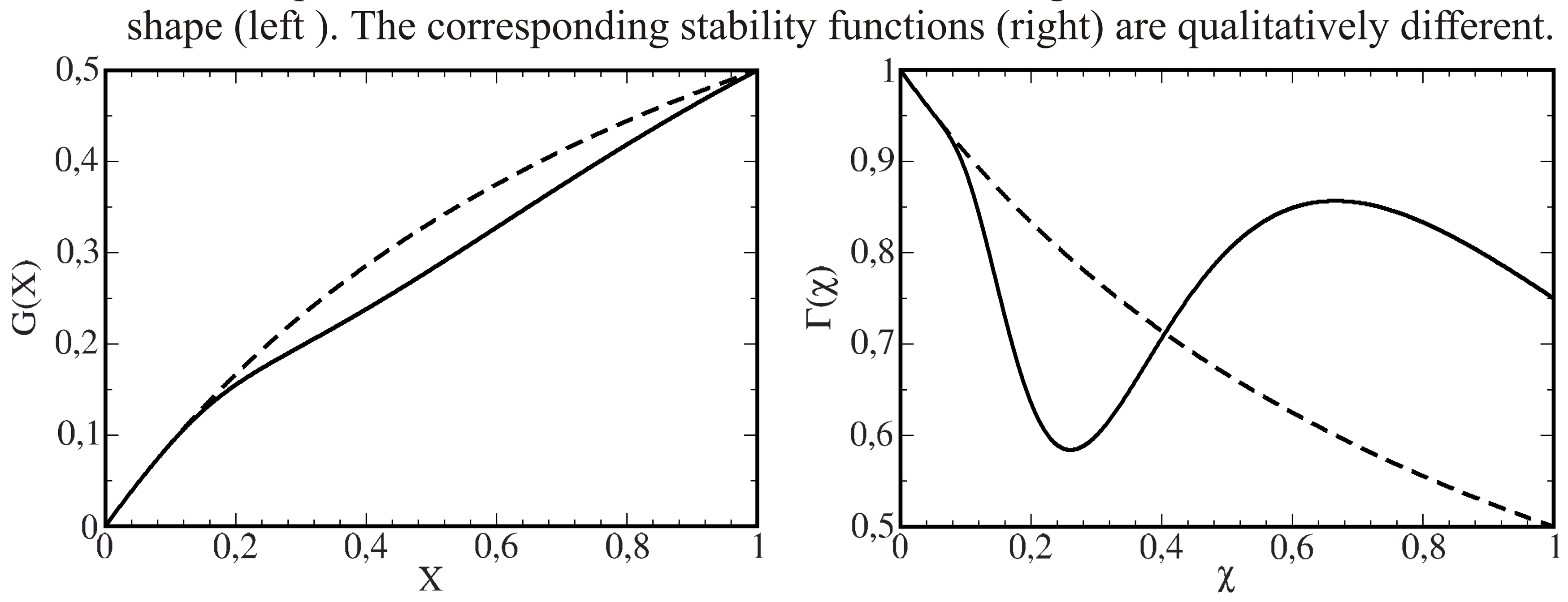
High sensitivity to prey abundance is found to be always stabilizing.

The effect of an interaction function on stability may be measured in terms of as a function of prey abundance

It is found that similar interaction functions correspond to qualitatively different stability functions.

Stability depends strongly on the exact form of interaction.

Below: Comparison of two interaction functions. Although the functions are similar in shape (left). The corresponding stability functions (right) are qualitatively different.



A general Foodchain Model

Consider a foodchain consisting of N species. The time evolution of the abundances $X_1 \dots X_N$ is given by

$$\dot{X}_n = \eta_n G_{n-1}(X_{n-1})X_n - G_n(X_n)X_{n+1} \qquad n = 1\dots N.$$

where $X_0 = X_0(X_1)$ is a Nutrient and X_{N+1} causes linear mortality of X_N. Every equilibrium state $X_1^* \dots X_N^*$ can be normalized by defining

$$x_n := \frac{X_n}{X_n{}^*}, \qquad g_n(x_n) := \frac{G_n(X_n)}{G_n(X_n{}^*)} = \frac{G_n(X_n{}^* x_n)}{G_n(X_n{}^*)} .$$

We obtain

$$\dot{x}_n = \alpha_n \left(g_{n-1}(x_{n-1})x_n - g_n(x_n)x_{n+1}\right)$$

The sensitivity to prey and nutrients can be defined as

$$\gamma_n := \frac{\partial}{\partial x_n} g_n(x_n)\bigg|_{\vec{x}=\vec{x}^*}, \quad h := -\frac{\partial}{\partial x_1} g_0(x_0(x_1))\bigg|_{\vec{x}=\vec{x}^*} .$$

If the species in the foodchain are similar, it is reasonable to assume

$$\Gamma := \gamma_1 = \dots = \gamma_{N-1}$$

Furthermore an alometric slowing down is observed in almost all foodchains. We model this universal behaviour by setting

$$\alpha_n = r^{n-1} \qquad n = 1\dots N$$

Our results do not depend on these assumptions. We can now write the Jacobian for arbitary N entirely in terms of r, h and